RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

FIRST YEAR

B.A./B.SC. SECOND SEMESTER (January – June), 2012

Mid-Semester Examination, March 2012

Date : 19/03/2012 **MATHEMATICS (Honours)**

Time : 11 am – 1 pm Paper : II Full Marks : 50

[Use separate Answer Books for each group]

Group – A

Answer Question No. 1 and any one from (2) & (3) :

1. Answer any one question : [1×5]

a) i) If $x-\alpha$ is a factor of f(x), prove that $f(\alpha)=0$.

ii) Define a multiple root. If α is a multiple root of f(x) = 0 of multiplicity r (r > 1), prove that α is a multiple root of $f'(x)=0$ of multiplicity r – 1. [2+(1+2)]

b) State Descarte's rule of signs. Use it to find the nature of roots of the equation, $x^n - 1 = 0$. [2+3]

2. a) If a, b, c, d be all real numbers greater than 1, prove that (a+1) (b+1) (c+1) (d+1) < 8(abcd +1). [3]

b) If $a_1, a_2, ..., a_n$; $b_1, b_2, ..., b_n$ be all real numbers then show that $\left(\sum_{i=1}^{n} a_i^2\right)\left(\sum_{i=1}^{n} b_i^2\right) \geq \left(\sum_{i=1}^{n} a_i b_i\right)^2$, the equality occurs when—

either i) $a_i = 0$ for i = 1, 2, ..., n or $b_i = 0$ for i = 1, 2, ..., n; or both $a_i = 0$ and $b_i = 0$ for i = 1, 2, ..., n.

or ii) $a_i = Kb_i$ for some nonzero real K, i = 1, 2, ..., n. [4]

3. a) If $a_1, a_2, ..., a_n$ be positive real numbers, not all equal, $p_1, p_2, ..., p_n$ be positive real numbers and m is rational, then show that

$\dfrac{p_1a_1^m + p_2a_2^m + ... + p_na_n^m}{p_1+p_2+...+p_n} >$ or $< \left(\dfrac{p_1a_1 + p_2a_2 + ... + p_na_n}{p_1+p_2+...+p_n}\right)^m$ according as m does not or does lie between 0 and 1. [4]

b) If a, b, c be three positive real numbers such that the sum of any two is greater than the third, prove that, abc > 8(s – a)(s – b)(s– c), where 2s = a + b + c. [3]

Answer Question No. 4 and any two from (5), (6) & (7) :

4. Answer any one question : [1×3]

a) Discuss the convergence of the series $\sum_{n=1}^{\infty} \dfrac{1}{n^p}$, $p > 0$. [3]

b) Test the convergence of the series

$1 + \dfrac{x}{1!} + \dfrac{2^2x^2}{2!} + \dfrac{3^3x^3}{3!} + ...$; x > 0. [3]

5. a) Let $\sum_{n=1}^{\infty} u_n$ be a series of positive real numbers and let $\overline{\lim}\dfrac{u_{n+1}}{u_n} = R$, $\underline{\lim}\dfrac{u_{n+1}}{u_n} = r$. Prove that $\sum_{n=1}^{\infty} u_n$ is convergent if R < 1 and $\sum_{n=1}^{\infty} u_n$ is divergent if r > 1. [3]

b) Test the convergence of the series $\left(\dfrac{2^2}{1^2} - \dfrac{2}{1}\right)^{-1} + \left(\dfrac{3^3}{2^3} - \dfrac{3}{2}\right)^{-2} + \left(\dfrac{4^4}{3^4} - \dfrac{4}{3}\right)^{-3} + ...$ [2]

6. a) Test the convergence of the series $\sum_{n=3}^{\infty} \frac{1}{n \log n (\log \log n)}$. [2½]

b) Prove that the series $\left(\frac{1}{2}\right)^p + \left(\frac{1 \cdot 3}{2 \cdot 4}\right)^p + \left(\frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}\right)^p + \ldots$ is convergent for p > 2 and divergent for $p \le 2$. [2½]

7. a) Prove that every absolutely convergent series of real numbers is convergent. [2½]

b) Examine the convergence of $1 - \frac{(2!)^2}{4!} + \frac{(3!)^2}{6!} - \ldots$ [2½]

Group – B

Answer any one from Question No. 8-9 and any one from Question No. 10-11 :

8. a) If $A = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$, show that $A^2 - 2A + I_2 = O$. Hence find A^{50}. [4]

b) Without expanding prove that : $\begin{vmatrix} a^3 & a^2 & 1 \\ b^3 & b^2 & 1 \\ c^3 & c^2 & 1 \end{vmatrix} = (ab + bc + ca) \begin{vmatrix} a^2 & a & 1 \\ b^2 & b & 1 \\ c^2 & c & 1 \end{vmatrix}$ [4]

c) Prove that : $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$ [5]

9. a) Express as the product of two determinants and hence prove that

$$\begin{vmatrix} (x-a)^2 & (x-b)^2 & (x-c)^2 \\ (y-a)^2 & (y-b)^2 & (y-c)^2 \\ (z-a)^2 & (z-b)^2 & (z-c)^2 \end{vmatrix} = 2(x-y)(y-z)(z-x)(a-b)(b-c)(c-a)$$ [4]

b) A is a non-singular matrix such that the sum of the elements in each row is K. Prove that the sum of the elements in each row of A^{-1} is $\frac{1}{K}$. [4]

c) If $(I+A)^{-1}(I–A)$ is a real orthogonal matrix, prove that the matrix A is skew-symmetric. [5]

10. a) Solve the following L.P.P. graphically :

Minimize $Z = 20x_1 + 10x_2$

subject to $x_1 + 2x_2 \le 40$

$3x_1 + x_2 \ge 30$

$4x_1 + 3x_2 \ge 60$

$x_1, x_2 \ge 0$ [3]

b) Using Charnes Big M-method, solve the L.P.P.

Maximize $Z = 5x_1 - 2x_2 + 3x_3$

subject to $2x_1 + 2x_2 - x_3 \ge 2$

$3x_1 - 4x_2 \le 3$

$x_2 + 3x_3 \le 5$

$x_1, x_2, x_3 \ge 0$ [6]

c) Prove that the intersection of two convex sets is also a convex set. [3]

11. a) $x_1 = 1$, $x_2 = 1$, $x_3 = 1$ and $x_4 = 0$ is a feasible solution of the system of equations

$$x_1 + 2x_2 + 4x_3 + x_4 = 7$$
$$2x_1 - x_2 + 3x_3 - 2x_4 = 4$$

Reduce the F.S. to B.F.S. [4]

b) Solve the following L.P.P. by simplex method :

Minimize $Z = 4x_1 + 8x_2 + 3x_3$

subject to

$$x_1 + x_2 \geq 2$$
$$2x_1 + x_3 \geq 5$$
$$x_1, \quad x_2, \quad x_3 \geq 0$$

[5]

c) If x_1, x_2 be real , show that the set given by $X = \{(x_1, x_2) \mid 9x_1^2 + 4x_2^2 \leq 36\}$ is a convex set. [3]